Subject Code: 241AN

Hall Ticket Number:

VR24

VIGNAN'S INSTITUTE OF MANAGEMENT AND TECHNOLOGY FOR WOMEN

(An Autonomous Institution)

I-B.Tech.–I-Semester Regular Examinations, February-2025

MATRICES AND CALCULUS

(CSE)

Time: 3 Hours **Max. Marks: 60**

(Answer All Questions)

Note: Question paper consists of Part-A & Part-B.

- **Part-A** for 10M, ii) **Part-B** for 50marks
- **Part A** is compulsory, consists of 10 sub questions from all units carrying equal marks.
- **Part-B** consists of **10 questions** (numbered from 2 to 11) carrying **10marks** each. From each unit there are 2 questions and the students should answer one of them. Hence the student should answer **5 questions** from **Part-B**.

PART- A

(10Marks)

1.a)	Define Echelon form of a Matrix	**1M**
1.b)	What is the rank of a unitary matrix of order 'n'	**1M**
1.c)	If the Eigen values of A are 1,1,2 then find the Eigen values of A^3	**1M**
1.d)	State Cayley-Hamilton Theorem.	**1M**
1.e)	Discuss the applicability of Rolle's mean value theorem for the function $f(x) = \|x\|$ in [-1, 1]	**1M**
1.f)	Define Gamma function	**1M**
1.g)	If u=$\frac{y}{x}, v$ =xy then find J($\frac{U,V}{X,Y}$)	**1M**
1.h)	Write the necessary conditions for f (x, y) to have a maximum or minimum at (a, b).	**1M**
1.i)	Evaluate$\int_{x=1}^{3}\int_{y=0}^{1} xy^2 dydx$	**1M**
1.j)	Skecth the area bounded by the curves y $= x, y = x^2$	**1M**

PART-B

(Answer any one full question from each unit. Each question carries 10 marks)

(50Marks)

2. a)	Find the rank of the matrix by reducing it to Normal form $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$	**5M**
b)	Investigating for what values of λ and μ do the system of equations x + y + z = 6, x + 2y + 3z = 10 , x + 2y + λz = μ have (i) no solution (ii) a unique solution (iii) An infinite number of solutions.	**5M**

OR

3. a)	Find the rank of a matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 4 & 8 & 7 & 5 \end{bmatrix}$ by reducing it to Echelon form.	**5M**
b)	Use Gauss Seidel iteration method to solve the system $10x + y + z = 12$, $2x + 10y + z = 13$, $2x + 2y + 10z = 14$.	**5M**
4. a)	Prove that if $\boldsymbol{\lambda}$ is an Eigen value of a non- singular matrix A then $\frac{\|A\|}{\lambda}$ is an eigen value of AdjA.	**2M**
b)	Reduce $x^2 - y^2 + 4z^2 + 4xy + 6xz + 2yz$ into canonical form by orthogonal transformations and also find its rank, index and signature	**8M**

OR

5. a) Find Eigen values and Eigen vectors of A= $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ **5M**

b) Verify Cayley-Hamilton theorem and hence find A^{-1} where

A= $\begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ **5M**

6. a) Verify Rolle's theorem for $f(x) = \log\left[\frac{x^2+ab}{x(a+b)}\right]$ in [a, b] **5M**

b) Show that $\Gamma(1/2) = \sqrt{\pi}$ **5M**

OR

7. a) Show that $1+ x < e^x < 1 + xe^x$, for $x > 0$ **5M**

b) Show that $\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$. where n is positive integer and m> -1 **5M**

8. a) Prove that u= x + y + z , v= xy+ yz+ zx ,w= $x^2 + y^2 + z^2$ are functional dependent and find the relation between them **5M**

b) A rectangular box open at the top is to have a volume 32 cubic feet. Find the dimensions of the box requiring least material for its construction **5M**

OR

9. a) If $u = \frac{x+y}{1-xy}$, $v = tan^{-1}x + tan^{-1}y$ then prove that u, v are functionally dependent and also write relation between them **5M**

b) Find three positive numbers whose number is 100 and whose product is maximum **5M**

10.a) Change the order of integration $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\, dx$ **5M**

b) Evaluate $\iint_R r^3 dr d\theta$ Over the area included between the circles r=2sinΘ and r=4sinΘ **5M**

OR

11.a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy$ **5M**

b) Evaluate $\iiint_V (xy + yz + zx) dx dy dz$. Where V is the region of space bounded by the planes x=0, x=1,y=0,y=2,z=0 and z=3 **5M**

*****VMTW*****